Ph.D. Thesis

Algebraic structures in some sets
of functions

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Abstract

This dissertation, entitled *Algebraic structures in some sets of functions*, is devoted to *algebrability* - a new trend in mathematical analysis which appeared about 15 years ago. It is a new concept to call a set **big** when it contains an algebraic structure (with large set of necessary generators) inside. This is the idea behind the following notions:

**Definition.** Let $\kappa$ be a cardinal number.

1. Let $\mathcal{L}$ be a linear commutative algebra and $A \subseteq \mathcal{L}$. We say that $A$ is $\kappa$-algebrable if $A \cup \{0\}$ contains a $\kappa$-generated algebra $B$ (i.e. the minimal cardinality of the system of generators of $B$ is $\kappa$);

2. Let $\mathcal{L}$ be a linear commutative algebra and $A \subseteq \mathcal{L}$. We say that $A$ is strongly $\kappa$-algebrable if $A \cup \{0\}$ contains a $\kappa$-generated algebra $B$ that is isomorphic to a free algebra.

Many papers concerning this notion were published and many examples were given, so we are mostly interested in the formulation of methods, which generalize some known and lead to new constructions. In the Ph.D. Thesis we describe two general methods: the method of *independent Bernstein sets* and of *exponential like functions* thanks to which we may prove several results in algebrability and strong algebrability of some sets in the algebras $\mathbb{R}^{\mathbb{R}}, \mathbb{C}^{\mathbb{C}}, \mathbb{R}^{\mathbb{N}}, C[0,1], \ell_{\infty}$ and present their applications. Most of the presented outcomes are the best possible, both in terms of complexity of the algebraic structure and the cardinality of the set of generators (usually it is the level of $c$ or $2^c$).